Quantum Mechanics. Introduction

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The fundamental diffence betn classical and quantum mechanics are as follows.

1. In classical mechanics the future history of a particle is completely determined by its initial position and momentum together with the force acting on the particle.

Quantum mechanics also arrives at relationships between observable quantities but the prediction or calculation of the final state is described only obeying the law of uncertainty.

(ii) The cause and effect relation are also hold in quantum mechanics but this is also obeying the limitation of uncertainity.

(iii) In classical mechanics x and p are position and momentum only and $xp - px = 0$
But in quantum mechanics $\hat{x}$ and $\hat{p}$ are quantum mechanical operators and here $\hat{x}\hat{p} - \hat{p}\hat{x} = i\hbar$
where $\hbar$ is $h/2\pi$ and h is plank constant.
As for example, the radius of the Bohr orbit in a ground state of H_2 atom is $5 \cdot 3 \times 10^{-11}$ m, but quantum mechanics states that it is the most probable radius.
And in different experiment the radius is experimentally observed a bit different from $5 \cdot 3 \times 10^{-11}$ m.

(Arthur Beiser)
(P.W. Atkins)

☐ Wave function and the physical significance of wave function :-

The probability that a particle will be ~~found~~ found at a given place in space at a certain instant of time is characterised by the function $\psi(x, y, z, t)$ in 3-D.
This is called the wave function.
ψ itself has no physical interpretation. The square

other quantities which may be obtained from wave function.

(Arthur Beiser)

☐ Normalization and Normalized wave function.

We have

$|\psi^*\psi|$ is proportional to the probability density ρ of finding the body described by ψ; the integral $\int \psi^*\psi \, dV$ over the space must be finite in order to have the particle somewhere in space.

If

$$\int_{-\infty}^{\infty} |\psi|^2 dV = 0,$$ means the particle doesnot exist

If $|\psi|^2 = \rho$ the probability density then

$$\int_{-\infty}^{\infty} |\psi|^2 dV = 1 \longrightarrow (1)$$ if the particle exists somewhere at all times then

$$\int_{-\infty}^{\infty} \rho \, dV = 1, \longrightarrow (1)$$

If the wave function following the above equation (1) then the wave function is called normalized.

If the wavefunction is not normalized, it converted to a normalized wavefunction

that $A\psi$ is a normalized wavefunction.

∴

Mathematically

$$\int_{-\infty}^{\infty} (A^*\psi^* A\psi)\, d\tau = 1$$

$$\Rightarrow A^2 \int_{-\infty}^{\infty} \psi^*\psi\, d\tau = 1$$

$$\therefore A = \left[\frac{1}{\int_{-\infty}^{\infty} |\psi^*\psi|\, d\tau}\right]$$

∴ A is called normalization Constant.

More about normalization:

(P. W. ATKINS)

Each and every solution of Schrodinger equation doesnot describe a possible matter wave associated with the particle. Only those solutions which satisfy general condition are admissible for the description of any physical system.

If we multiply the solution of a Schrodinger equation by a Const and if the resulting function is still a solution of the quantum mechanical system,

If ψ is a solution of the Q.M system and $N\psi = \psi'$ is also a solution, then

$$\int_{-\infty}^{\infty} |\psi'^{*} \psi'| \, d\tau = N^2,$$ the N^2 is called the norm of the wavefunction ψ

The wave function with unit norm is called normalized.

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Fundamental Postulate Of Quantum Mechanics:

The fundamental postulate of quantum mechanics is composed of two postulates.

(a) Representation of states (b) Expectation value and observables.

(a) The state of a quantum mechanical system is described or represented by wavefunction $\psi(x,t)$. The wavefunction contains all information which nature permits one to give about the state of the system at any instant of time. Actually ψ means the physical concept described by the mathematical function $\psi(x,t)$. The wavefunction is not directly measurable. Actually all constant multiples of given ψ, say $c\psi$ describes the same quantum state (c is a scalar)

If ψ_1 and ψ_2 are wavefunctions describing any two states of a given system, then corresponding to every linear combination of the two wavefunction, $\psi = (c_1\psi_1 + c_2\psi_2)$ there exists a state of the system.

This is applicable for the linear combination of multiple number of wavefunction. i.e

$$\psi = c_1\psi_1 + c_2\psi_2 + c_3\psi_3 + \cdots$$

(b) Expectation value and observable :-

Each dynamical variable in quantum mechanics is represented by a linear operator.

$$A_{op} = A(X_{op}, P_{op})$$
$$= A(X, -i\hbar\nabla)$$

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D Operator.

The operator act on the wavefunction of the system. The effect of an operator A on a wave function ψ is to convert it to another wave function denoted by $A\psi$.

Linearity of the operator means that a linear combination of two or more wave functions say ψ_1 and ψ_2 is converted into the same linear combination of $A\psi_1$ and $A\psi_2$

$$A(C_1\psi_1 + C_2\psi_2) = C_1(A\psi_1) + C_2(A\psi_2)$$

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